Journal of Engineering Physics and Thermophysics, Vol. 78, No. 1, 2005

ASSESSMENT OF OPERATION OF AN UNDERGROUND CLOSED-LOOP GEOTHERMAL HEAT EXCHANGER

W. Nowak and A. A. Stachel UDC 532.546

The results of calculations that allow one to obtain the characteristics of a closed-loop geothermal heat exchanger (C-LGHE) on the basis of our own approximate mathematical model are presented. These results are given in the form of appropriate tables containing the reduced quantities characterizing operation of such a geothermal heat exchanger. On this basis, C-LGHE operation is analyzed and conclusions on the possibilities of utilization of geothermal energy in binary power stations are drawn.

The results of analysis of the technical and economic possibilities of construction of a closed-loop geothermal heat exchanger (C-LGHE) are presented. Exchangers of this type can form one of the elements of geothermal binary power stations utilizing high-temperature energy accumulated in rocks at large depths. In the context of a lack of data on thermal calculations of C-LGHEs, we performed calculations that allowed us to obtain the characteristics of these heat exchangers by our own approximate mathematical model [7].

Introduction. Two fundamental designs of extracting installations can be used in acquisition of geothermal energy, namely, so-called open or closed systems. A research program related to this topic is underway at Berlin Technical University [11, 12], dealing with the possibilities and cost effectiveness of the construction of an underground closed-loop geothermal heat exchanger (C-LGHE). This C-LGHE consists of a set of underground pipelines, where a liquid heat carrier is pumped for acquisition of geothermal energy from the rock mass and transfer of it to the binary power station (Fig. 1).

The amount of acquired geothermal energy depends on the area of the heat-transfer surface of the heat exchanger, the rate of liquid flow, and the depth of the heat-exchanger location, which is greatly related to the temperature of the surrounding rocks.

In order to determine temperature in the case of transient heat conduction in the surrounding rock bed, we can use the solutions of heat-conduction equations, which are widely provided in the literature for various boundary conditions. Some of the solutions for averaged thermal and physical properties of the rock bed enable one to determine time-dependent reduced linear thermal resistance in the bed. For example, in [3, 4] a procedure is presented for calculation of linear thermal resistance of the rock bed through the introduction of a time-dependent radius of interaction of the surrounding medium $r_s = f(t)$ under thermal conditions in the rock bed.

According to [1, 2], if $r_s >> r_w$, the radius r_s increasing with time t can be determined from the relation

$$r_s = 2\sqrt{a_s t}\,, \tag{1}$$

where r_w is the radius of the borehole in the rock and a_s is the thermal diffusivity of the rock bed. The overall heat-transfer coefficient k_m which corresponds to $t = t_m$ can be determined from the equations given by Charnyi [1, 2]:

$$\frac{1}{k_m} = \frac{1}{\alpha} + \frac{D_1}{2}\sum_{i=1}^{n}\frac{1}{\lambda_i}\ln\frac{D_{i+1}}{D_i} + \frac{D_1}{2\lambda_s}\ln\frac{4\sqrt{a_s t_m}}{D_{n+1}} \tag{2}$$

or

Technical University of Szczacian, Department of Heat Engineering, Al. Piastów 19, 70-310, Szczacian, Poland; email: andrzej.stachel@ps.pl. Published in Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 1, pp. 131–137, January–February, 2005. Original article submitted September 21, 2004.

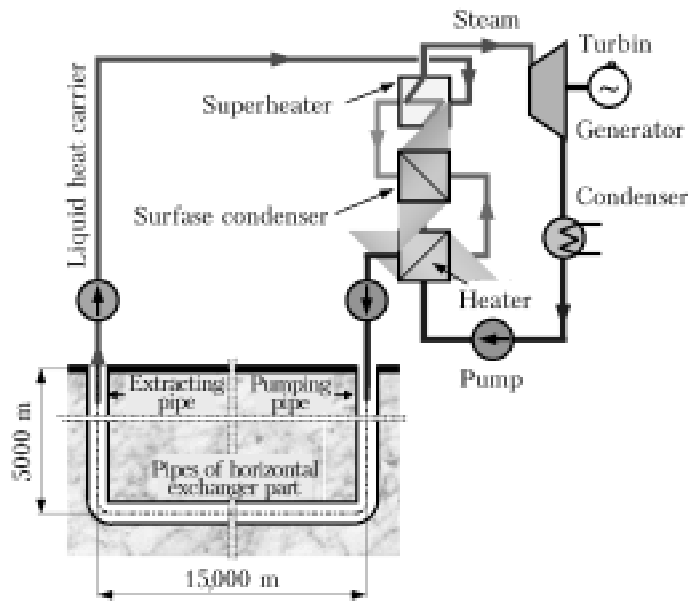

Fig. 1. Schematic of an underground closed-loop geothermal heat exchanger co-operating with the binary power station.

$$\frac{1}{k_m} \cong \frac{1}{\alpha} + \frac{D_1}{2\lambda_s} \ln \frac{4\sqrt{a_s t_m}}{D_1}, \tag{2a}$$

where relation (2a) corresponds to the case of deep wells, when the piping thermal resistance can be neglected.

Another way of calculating the overall heat-transfer coefficient is provided by Dyad'kin and Gendler [5], who derived the following formulas from the analytical solutions presented in [6, 10]:

$$k_m = \frac{k_z}{1 + \text{Bi} \ln (1 + \sqrt{\bar{\gamma}\,\text{Fo}_m})}, \tag{3}$$

where $\text{Bi} = \alpha l / \lambda_s$ and $\text{Fo} = a_s t / l^2$,

$$\frac{1}{k_z} = \frac{1}{\alpha} + \frac{D_1}{2} \sum_{i=1}^{n} \frac{1}{\lambda_i} \ln \frac{D_{i+1}}{D_i}. \tag{4}$$

The parameter $\bar{\gamma}$ in Eq. (3) depends on the value of the Biot number. If $\text{Bi} \to \infty$, which corresponds practically to $\text{Bi} > 30$, $\bar{\gamma} = \pi$; in the remaining cases $\bar{\gamma} = 2$. Then, for $\text{Bi} > 30$ Eq. (3) can be written as follows:

$$k_m = \frac{k_z}{1 + \text{Bi} \ln (1 + \sqrt{\pi\,\text{Fo}_m})}. \tag{3a}$$

For $\text{Bi} \to \infty$ relation (3a) takes the form

$$k_m \cong \frac{\lambda_s}{r_1 \ln (1 + \sqrt{\pi\,\text{Fo}_m})}.$$

The C-LGHE is independent of its localization and is characterized by long-term operation at a relatively small power of the circulation pumps pumping liquid through a tight system of underground heat exchangers. An important feature of the C-LGHE is its ability for flexible operation. Such heat exchangers enable one to get geothermal energy in all locations where open systems cannot be applied for geological reasons.

However, a disadvantage of the C-LGHE is its significant investment cost as well as the necessity of mastering technological processes related to drilling and installation of tight pipeline systems at large depths. Drilling techniques allowing extraction of hydrocarbon resources at large distances from the platform can be used in drilling of the holes. This is related to the utilization of drilling robots with application of adequate control for temperatures not exceeding 175^{o}C. The existing practical possibilities regard utilization of boreholes of large depths (up to 5000 m) and a horizontal length up to 15,000 m, which was discussed in [12].

This paper presents some results of the numerical calculations of the C-LGHE and the results of calculations obtained using the analytical calculation model of [7], where the above-reported relations were applied to calculation of the time-dependent overall heat-transfer resistance or the overall coefficient of heat transfer from the rock bed to the heat carrier.

Temperature Field in an Underground Closed-Loop Geothermal Heat Exchanger. Using the above-presented relations for the overall coefficient of heat transfer between the bed and the heat carrier, we developed a relatively simple calculation model of a C-LGHE [7]. In so doing we applied the following simplification assumptions:

• The heat exchanger is divided into three constituent elements connected in series and includes a vertical pumping pipe, a horizontal pipe located at some depth and forming a principal part of the heat exchanger, and a vertical extracting pipe. For each of the above-mentioned elements, a somewhat different calculation model is used, due to different heat-transfer conditions. In all cases, the energy-balance equation for an elementary heat-transfer surface is used. A common feature is the assumption that the overall coefficient of heat transfer from the bed to the heat carrier is known and the same for all three elements of the heat exchanger, which is time-dependent and can be determined from (2) or (3).

• In the case of extracting and pumping pipes, the bed temperature at a significant distance from these elements of the heat exchanger varies linearly with depth according to the relation [9]

$$T_{\mathrm{s}}(h) = a + bh \,. \tag{5}$$

The temperature of the rock bed surrounding a horizontal part of the heat exchanger depends on the depth H and can be determined from (5):

$$T_{\mathrm{s}H} = T_{\mathrm{s}}\big|_{h=H} = a + bH \,. \tag{5a}$$

Due to the constant temperature of the surrounding bed, the horizontal element of the heat exchanger can be considered as a heat exchanger of the "condenser" type.

• Elementary heat power transferred from the bed to the heat carrier in the heat exchanger can be determined through the overall heat-transfer coefficient k_{m}:

$$d\dot{Q} = k_m \,[T_{\mathrm{s}}(h) - T_{\mathrm{p}}(h)]\, dA \,. \tag{6}$$

• Elementary heat power taken up by the heat carrier is described as

$$d\dot{Q} = l_i \dot{W}_i dT_{\mathrm{p}i} \,, \tag{7}$$

where l_i for the subsequent elements of the heat exchanger (i = 1, 2, 3) assumes the following values: l_1 = +1 for the pumping pipe, l_2 = +1 for the horizontal pipes, and l_3 = –1 for the vertical extracting pipe.

Applying the relations describing the overall coefficient of heat transfer between the bed and the fluid and taking into account the above-listed simplifications, in what follows we will use a theory of an underground closed-loop geothermal heat exchanger that will enable us to determine analytically an approximate temperature field of the component removing heat from the surrounding bed [7].

For each of the heat-exchanger elements, the conditions of heat exchange between the bed and the heat carrier can be described by an ordinary differential equation of the first order

$$\frac{d\vartheta_i}{dh_i} + \alpha_i \vartheta_i + \beta_i = 0 \quad \text{at} \quad i = 1, 2, 3, \tag{8}$$

with a general solution in the form

$$\vartheta_i = C_i \exp(-\alpha_i h_i) - \frac{\beta_i}{\alpha_i} . \tag{9}$$

The integration constant C_i can be determined from the following boundary condition at the inlet to each element of the heat exchanger: at $h_i = 0$, $\vartheta_i(0) = \vartheta_{i0}$. The coefficients α_i and β_i in Eq. (8) for a particular heat-exchanger element (i = 1, 2, 3) have different values and are presented in [7]. Without going into details of the derivations and transformations, which are also given in [7], the relations describing the temperature field T_p, the temperature difference ϑ_i, and the reduced temperature difference Θ_i can be derived for each subsequent element of the heat exchanger.

For example, the temperature differences ϑ_i for specific elements of the heat exchanger are as follows:

- for element 1

$$\vartheta_1(h_1) = -\left[(T_{p10} - T_{s0}) + \frac{T_{sH} - T_{s0}}{kN_1}\right] \exp(-kN_1\bar{h}_1) + \frac{T_{sH} - T_{s0}}{kN_1} ; \tag{10}$$

- for element 2

$$\vartheta_2(l) = T_{sH} - T_{p2}(l) = (T_{sH} - T_{p1H}) \exp(-kN_2\bar{l}) ; \tag{11}$$

- for element 3

$$\vartheta_3(h_3) = \left[(T_{p2L} - T_{sH}) - \frac{T_{sH} - T_{s0}}{kN_3}\right] \exp(-kN_3\bar{h}_3) + \frac{T_{sH} - T_{s0}}{kN_3} . \tag{12}$$

Here $k = k_m / k_1$ and $N_i = kA_i / \dot{W}$ is the number of transfer units.

The reduced temperature differences at the outlet from the particular element of the heat exchanger can be determined from (10)–(12) as

$$\Theta_{1H} = \frac{T_{sH} - T_{p1H}}{T_{sH} - T_{p10}} = \frac{\phi}{kN_1} + \left[1 - \phi\left(1 + \frac{1}{kN_1}\right)\right] \exp(-kN_1) , \tag{13}$$

$$\Theta_{2L} = \frac{T_{sH} - T_{p2L}}{T_{sH} - T_{p1H}} = \exp(-kN_2) , \tag{14}$$

$$\Theta_{3H} = \frac{T_{sH} - T_{p3H}}{T_{sH} - T_{p2L}} = \frac{\phi}{\psi} + \exp(-kN_3) - \frac{\phi}{\psi kN_3}[1 - \exp(-kN_3)] , \tag{15}$$

where $\Psi = \Theta_{1H}\Theta_{2L}$, $\Phi = \dfrac{T_{sH} - T_{s0}}{T_{sH} - T_{p10}}$.

In order to analyze the influence of all quantities characterizing subsequent heat-exchanger elements on the reduced temperature difference at the outlet from the third element, we introduce the following relation:

$$\Theta_{1H,2L,3H} = \Theta_{1H}\Theta_{2L}\Theta_{3H} = \Theta_{1H,2L}\Theta_{3H} , \tag{16}$$

where

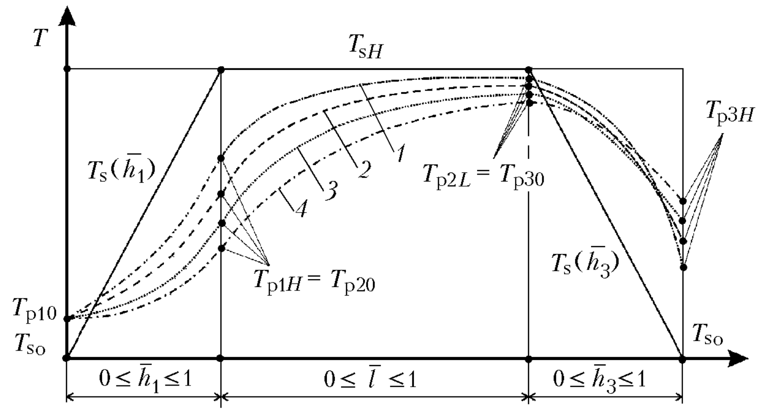

Fig. 2. Fluid temperature fields in C-LGHE for different numbers of transfer units ($N_4 < N_3 < N_2 < N_1$ and N_i corresponds to the ith curve).

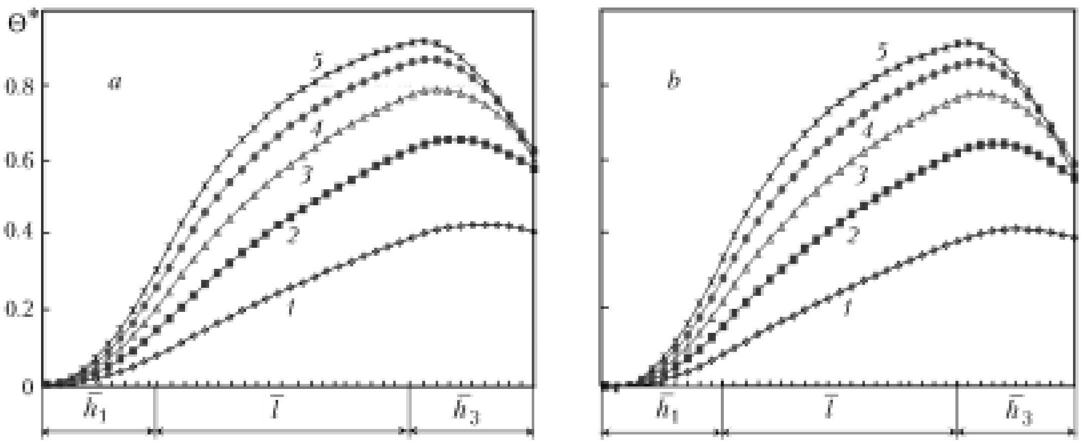

Fig. 3. Reduced temperature fields for three C-LGHE elements with different numbers of transfer units: a) k = 1, Φ = 1, N = 0.2 (1), 0.4 (2), 0.6 (3), 0.8 (4), 1 (5); b) k = 0.5, Φ = 1.1, N = 0.4 (1), 0.8 (2), 1.2 (3), 1.6 (4), 2 (5).

$$\Theta_{1H,2L} = \Theta_{1H}\Theta_{2L} = \psi = \left\{(1-\phi)\exp(-kN_1) + \frac{\phi}{N_1}[1-\exp(-kN_1)]\right\}\exp(-kN_2)\,. \quad (17)$$

Then the relation describing the adequately defined temperature difference at the outlet from the third heat-exchanger element with consideration of the fluid temperature effect at the inlet to the first element assumes the form

$$\Theta_{1H,2L,3H} = \frac{T_{sH} - T_{p3H}}{T_{sH} - T_{p10}} = \phi + \Theta_{1H,2L}\exp(-kN_3) - \frac{\phi}{N_3}[1-\exp(-kN_3)]\,. \quad (18)$$

A general view of the curves illustrating the temperature fields for subsequent elements of the heat exchanger with different numbers of heat-transfer units is presented in Fig. 2.

Calculation Results. With the above-presented relations, we carry out simple calculations of the reduced temperature differences. Figure 3 shows the reduced temperature fields Θ_1, Θ_2, and Θ_3 for three elements of the C-LGHE as functions of the reduced length for different numbers of heat-transfer units N and the given values of k and Φ with $\Theta_i^* = 1 - \Theta_i$. Figure 4 presents the reduced temperature fields of the first and second elements of the C-LGHE for different values of the reduced heat-transfer coefficient with given N and Φ.

The remaining results of calculations for the entire C-LGHE, which are the reduced temperature differences at the outlet, are presented in Tables 1 and 2. These tables show the influence of important characteristics of the C-LGHE on the temperature of the heat carrier flowing out from the heat exchanger (this temperature is equal to the

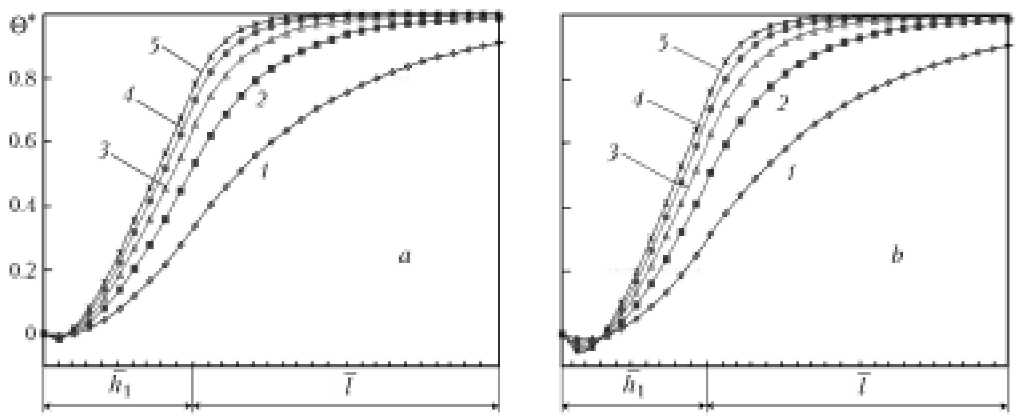

Fig. 4. Reduced temperature fields for two C-LGHE elements with different values of the reduced heat-transfer coefficient: a) Φ = 1.1, N = 1, k = 1 (1), 2 (2), 3 (3), 4 (4), 5 (5); b) Φ = 1.2, N = 2, k = 0.5 (1), 1 (2), 1.5 (3), 2 (4), 2.5 (5).

TABLE 1. Reduced Temperature $\Theta_{1H,2L,3H}$ of the Heat Carrier at the Outlet from the C-LGHE

Φ	N_1				
	0.2	0.4	0.6	0.8	1.0
0.9	0.577	0.402	0.344	0.341	0.361
1.0	0.591	0.424	0.372	0.374	0.399
1.1	0.605	0.446	0.401	0.407	0.438
1.2	0.619	0.469	0.429	0.441	0.476
1.3	0.634	0.491	0.457	0.525	0.514

TABLE 2. Reduced Temperature $\Theta_{1H,2L,3H}$ of the Heat Carrier at the Outlet from the C-LGHE with Insulation of the Outlet Socket

Φ	N_1				
	0.2	0.4	0.6	0.8	1.0
0.9	0.602	0.364	0.220	0.134	0.082
1.0	0.608	0.370	0.227	0.139	0.086
1.1	0.613	0.377	0.233	0.144	0.089
1.2	0.619	0.384	0.239	0.149	0.093
1.3	0.625	0.391	0.245	0.154	0.096

temperature at the inlet to the power-station system). Of significance in the influence on temperature T_{pd} are the conditions of heat transfer in the third heat-exchanger element. In the case of its insulation, the efficiency of the C-LGHE increases, which is seen from Table 2. On the other hand, the temperature at the outlet from the system T_{pw} is closely related to the way of its utilization in the power station.

The amount of geothermal energy acquired by the thermal power station is affected by the heat-carrier temperature at the inlet to (T_{pd}) and the outlet from the system (T_{pw}) and the heat-carrier flow rate in accordance with the expression

$$\dot{Q} = \dot{m}c_p\,(T_{pd} - T_{pw}) = \dot{W}\,(T_{pd} - T_{pw})\ . \tag{19}$$

The heat-carrier temperature at the inlet to the system T_{pd} depends on the operational efficiency of the C-LGHE. If the overall efficiency of the power station is known, its power can be determined from the relation

$$N_e = \dot{Q}\eta_e\,. \tag{20}$$

The overall efficiency of the power station is related to the efficiency of the comparative Clausius–Rankine cycle η_{CR}, which depends on the temperature attained in the heat exchanger and on the ambient temperature.

Taking into account the fact that the drilling technique now in application, which utilizes robots with adequate control, cannot be realized under temperatures exceeding 175^{o}C [11], this temperature must be assumed as the upper limit of the heat-carrier temperature. In this connection, water can be used as a working medium in the Clausius–Rankine cycle within the temperature range 120–170^{o}C. For temperatures lower than 120^{o}C, fluid with a low boiling point should be used as a heat carrier.

CONCLUSIONS

On the basis of the above-presented considerations and the conducted analysis, which is largely omitted due to space restrictions, we can make the following concluding remarks:

• The temperature field of the heat carrier in C-LGHE depends on the heat-carrier flow rate and reduced overall heat-transfer coefficient.

• In order to attain an adequate heat-carrier temperature at the outlet from the vertical hole, the heat-transfer surface area of a horizontal part with respect to the surface area of vertical holes must be such that the heat carrier at the outlet from the horizontal part could reach the rock-bed temperature and the decrease in the temperature in the extracting well must be as low as possible. The best design would be with insulation of the extraction well.

• In order to account for the decreasing effect of the overall heat-transfer coefficient, C-LGHE must be designed such that the heat carrier could attain the maximum rock-bed temperature for a longer period of operation.

• Because the temperature of hot water at the surface of the extracting well (at the inlet to the heat exchanger) usually does not exceed 160^{o}C, two-component systems can be used. Water or ecological low-boiling point refrigerants [8] can be used as working media in the secondary loop.

• In order to improve the efficiency of binary power stations, designs incorporating additional superheating or combination with a gas turbine would be advised.

NOTATION

A, area, m^2; a, thermal diffusivity, m^2/sec; Bi, Biot number; c_p, specific heat at constant pressure, J/(kg·K); D, diameter, m; Fo, Fourier number; H, depth of heat-exchanger location, m; h, depth, m; k, reduced heat-transfer coefficient; k_m, overall heat-transfer coefficient, W/(m^2·K); L, length of the heat exchanger, m; l, length, m; $\overline{l}$, reduced length; $\dot{m}$, mass flow rate, kg/sec; N, number of transfer units; $\dot{Q}$, heat power, W; r, radius, m; T, temperature, ^{o}C; t, time, sec; W, heat capacity, J/K; α, heat-transfer coefficient, W/(m^2·K); η, efficiency; λ, thermal conductivity, W/(m·K); Θ, reduced temperature difference; ϑ, temperature difference, ^{o}C. Subscripts: d, inlet; p, fluid; s, rock; 1, 2, and 3, subsequent elements of the heat exchanger.

REFERENCES

1. I. A. Charnyi, Movement of the boundary of change in aggregate state with body cooling or heating, Izv. OTN AN SSSR, No. 2 (1948).
2. I. A. Charnyi, Heating of a critical area of formation in pumping of hot water into a well, Neft. Khoz., No. 3 (1953).
3. E. B. Chekalyuk, *Thermodynamics of the Oil-Bearing Bed* [in Russian], Nedra, Moscow (1965).
4. G. A. Cheremenskii, *Geothermy* [in Russian], Nedra, Leningrad (1972).
5. Yu. D. Dyad'kin and S. G. Gendler, *Heat and Mass Transfer Processes in Extraction of Geothermal Energy* [in Russian], Izd. LGI, Leningrad (1985).
6. Yu. D. Dyad'kin, Yu. B. Shuvalov, and S. G. Gendler, *Thermal Processes in Excavations* [in Russian], Izd. LGI, Leningrad (1978).

7. W. Nowak, Theory of an Underground Closed-Loop Geothermal Heat Exchanger [in Polish], TU of Szczecin, 2003 (nonpublished internal report of Department of Heat Engineering).
8. W. Nowak and A. Borsukiewicz-Gozdur, Duales Heizwerk gespeist mit einem Geothermalen wasser einer mittlern Enthalpie, 1. Fachkongress Geothermischer Strom, 12—13.11.2003, Neustadt Glewe.
9. W. Nowak, R. Sobański, M. Kabat, and T. Kujawa, *Systems of Acquisition and Utilization of Geothermal Energy* [in Polish], TU of Szczecin Publishers, Szczecin (2000).
10. M. A. Pudovkin, A. I. Salamatin, and V. A. Chugunov, *Temperature Processes in Operating Wells* [in Russian], Izd. KGU, Kazan' (1977).
11. H. Wolff, F. Möller, T. Besser, S. Schmidt, J. Oppelt, and J. Treviranus, Ansätze fortschrittlichen Bohr- und Komplettierungstechnik für die Errichtung eines Unterträging Geschlossenen Geothermischen Wärmetauscher, Projekt "Untertägig Geschlossener Geothermischer Wärmetauscher" BMU-ZIP 0327506, 20 Jahre Tiefe Geothermie in Deutschland, 7. Geothermische Fachtagung, 06-08.11.2002, Waren.
12. H. Wolff, S. Schmidt, F. Möller, B. Legarth, J. Oppelt, and J. Treviranus, Geothermische Stromerzeugung, Projekt "Untertägig Geschlossener Geothermischer Wärmetauscher" BMU - ZIP 0327506, Status-Quo, Juni 2002, Vortrag, Symposium "Geothermische Stromerzeugung," Landau 20/21 Juni 2002.